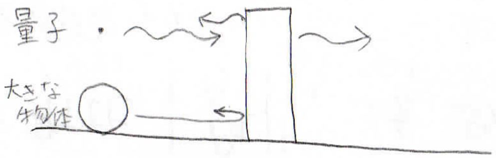
トンネル効果

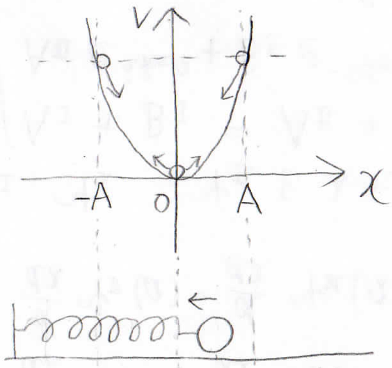
☆ 言葉での表現

量子は ある確率でポテンシャルの障壁を透過することができる



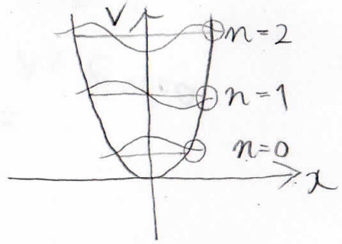
☆ 調和振動子

・マクロ



$|x| > A$ の領域に物体は存在し得ない.

・ミクロ



$E = \left(n + \frac{1}{2}\right)\hbar\omega$

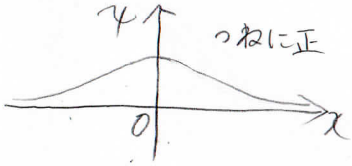
$V = \frac{m\omega^2}{2}x^2$

$\psi(x) \propto H_n(\xi)\exp\left(-\frac{\xi^2}{2}\right)$

$n=0$ のとき,

$\psi(x) \propto \exp\left(-\frac{1}{2}\alpha^2 x^2\right)$

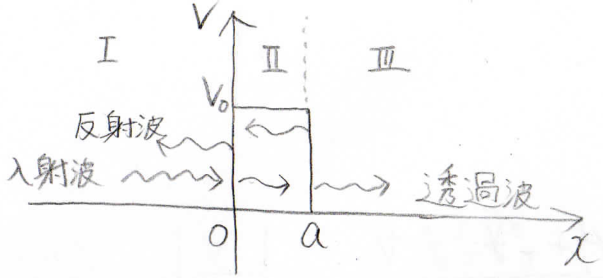
(ただし, $\alpha = \frac{m\omega}{\hbar}$)



$|\psi|^2 > 0$

☆ ポテンシャル障壁透過確率

・計算モデル



領域I

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + 0\right)\psi_I = E\psi_I$$

$k_I = \frac{\sqrt{2mE}}{\hbar}$ とおく

$$\psi_I = \underbrace{A_I e^{ik_I x}}_{\text{正方向に進む}} + \underbrace{B_I e^{-ik_I x}}_{\text{負方向に進む}}$$

正方向に進む → 入射波　　負方向に進む → 反射波

領域II

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_0\right)\psi_{II} = E\psi_{II}$$

$k_{II} = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$ とおく

$$\psi_{II} = A_{II} e^{ik_{II} x} + B_{II} e^{ik_{II} x}$$

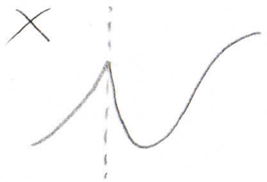
領域III

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + 0\right)\psi_{III} = E\psi_{III}$$

$k_{III} = k_I = \frac{\sqrt{2mE}}{\hbar}$

$\psi_{III} = A_{III} e^{ik_I x}$　反射波なし

・境界条件



滑らかに繋がるはず

$$
\begin{cases}
\psi_{\mathrm{I}}(0) = \psi_{\mathrm{II}}(0) \\
\psi_{\mathrm{II}}(a) = \psi_{\mathrm{III}}(a) \\
\dfrac{d}{dx}\psi_{\mathrm{I}}(0) = \dfrac{d}{dx}\psi_{\mathrm{II}}(0) \\
\dfrac{d}{dx}\psi_{\mathrm{II}}(a) = \dfrac{d}{dx}\psi_{\mathrm{III}}(a)
\end{cases}
$$

$\psi_{\mathrm{I}}, \psi_{\mathrm{II}}, \psi_{\mathrm{III}}$ を入れると

$$
\begin{cases}
A_{\mathrm{I}} + B_{\mathrm{I}} = A_{\mathrm{II}} + B_{\mathrm{II}} & \cdots ① \\
A_{\mathrm{II}} e^{ik_{\mathrm{II}}a} + B_{\mathrm{II}} e^{-ik_{\mathrm{II}}a} = A_{\mathrm{III}} e^{ik_{\mathrm{I}}a} & \cdots ② \\
ik_{\mathrm{I}}(A_{\mathrm{I}} - B_{\mathrm{I}}) = ik_{\mathrm{II}}(A_{\mathrm{II}} - B_{\mathrm{II}}) & \cdots ③ \\
ik_{\mathrm{II}}(A_{\mathrm{II}} e^{ik_{\mathrm{II}}a} - B_{\mathrm{II}} e^{-ik_{\mathrm{II}}a}) = ik_{\mathrm{I}} A_{\mathrm{III}} e^{ik_{\mathrm{I}}a} & \cdots ④
\end{cases}
$$

反射比 $\left|\dfrac{B_{\mathrm{I}}}{A_{\mathrm{I}}}\right|^2$，透過比 $\left|\dfrac{A_{\mathrm{III}}}{A_{\mathrm{I}}}\right|^2$ を求める

②,④より

$$ik_{\mathrm{II}}(A_{\mathrm{II}} e^{ik_{\mathrm{II}}a} - B_{\mathrm{II}} e^{-ik_{\mathrm{II}}a}) = ik_{\mathrm{I}}(A_{\mathrm{II}} e^{ik_{\mathrm{II}}a} + B_{\mathrm{II}} e^{-ik_{\mathrm{II}}a})$$

$$(k_{\mathrm{II}} - k_{\mathrm{I}}) A_{\mathrm{II}} e^{ik_{\mathrm{II}}a} = (k_{\mathrm{II}} + k_{\mathrm{I}}) B_{\mathrm{II}} e^{-ik_{\mathrm{II}}a}$$

$$A_{\mathrm{II}} = \left(\frac{k_{\mathrm{II}} + k_{\mathrm{I}}}{k_{\mathrm{II}} - k_{\mathrm{I}}} e^{-2ik_{\mathrm{II}}a}\right) B_{\mathrm{II}}$$

①より

$$A_{\mathrm{I}} + B_{\mathrm{I}} = \left(\frac{k_{\mathrm{II}} + k_{\mathrm{I}}}{k_{\mathrm{II}} - k_{\mathrm{I}}} e^{-2ik_{\mathrm{II}}a} + 1\right) B_{\mathrm{II}}$$

$$B_{\mathrm{II}} = \frac{A_{\mathrm{I}} + B_{\mathrm{I}}}{\dfrac{k_{\mathrm{II}} + k_{\mathrm{I}}}{k_{\mathrm{II}} - k_{\mathrm{I}}} e^{-2ik_{\mathrm{II}}a} + 1}$$

③に代入

$$A_{\mathrm{I}} - B_{\mathrm{I}} = \frac{k_{\mathrm{II}}}{k_{\mathrm{I}}} \frac{\dfrac{k_{\mathrm{II}} + k_{\mathrm{I}}}{k_{\mathrm{II}} - k_{\mathrm{I}}} e^{-2ik_{\mathrm{II}}a} - 1}{\dfrac{k_{\mathrm{II}} + k_{\mathrm{I}}}{k_{\mathrm{II}} - k_{\mathrm{I}}} e^{-2ik_{\mathrm{II}}a} + 1} (A_{\mathrm{I}} + B_{\mathrm{I}})$$

$$\frac{B_{\mathrm{I}}}{A_{\mathrm{I}}} = \frac{(k_{\mathrm{I}}^2 - k_{\mathrm{II}}^2)(1 - e^{2ik_{\mathrm{II}}a})}{(k_{\mathrm{I}} + k_{\mathrm{II}})^2 - (k_{\mathrm{I}} - k_{\mathrm{II}})^2 e^{2ik_{\mathrm{II}}a}}$$

反射比 $\left|\dfrac{B_{\mathrm{I}}}{A_{\mathrm{I}}}\right|^2 = \dfrac{(k_{\mathrm{I}}^2 - k_{\mathrm{II}}^2)^2 \sin^2 k_{\mathrm{II}}a}{4k_{\mathrm{I}}^2 k_{\mathrm{II}}^2 + (k_{\mathrm{I}}^2 - k_{\mathrm{II}}^2)^2 \sin^2 k_{\mathrm{II}}a}$

$\left|\dfrac{B_{\mathrm{I}}}{A_{\mathrm{I}}}\right|^2 + \left|\dfrac{A_{\mathrm{III}}}{A_{\mathrm{I}}}\right|^2 = 1$ より

$$\left|\frac{A_{\mathrm{III}}}{A_{\mathrm{I}}}\right|^2 = \frac{4k_{\mathrm{I}}^2 k_{\mathrm{II}}^2}{4k_{\mathrm{I}}^2 k_{\mathrm{II}}^2 + (k_{\mathrm{I}}^2 - k_{\mathrm{II}}^2)^2 \sin^2 k_{\mathrm{II}}a}$$