等温変化、断熱変化

※理想気体とする

$$pV = nRT$$

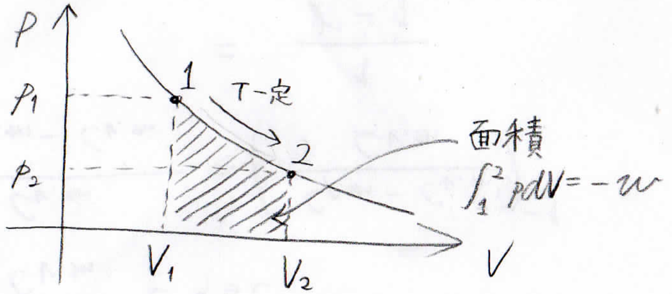
$dU = C_v dT$ (C_vの温度依存性考えない)

☆等温変化　T一定 ($\Delta T = 0$)

$$\Delta U = \int C_v dT = C_v \Delta T = 0$$

$$\Delta U = q + w = 0$$

$$\begin{cases} dw = -pdV \\ dq = pdV \end{cases}$$



$$w = -\int_1^2 pdV$$

$$= -\int_{V_1}^{V_2} \frac{nRT}{V} dV$$

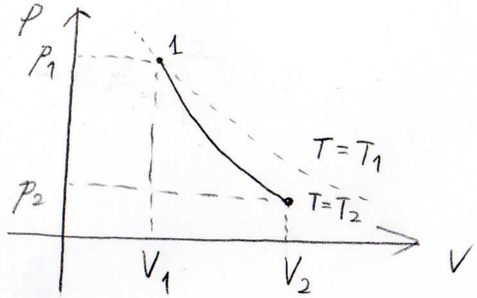
$$= -nRT \ln\left(\frac{V_2}{V_1}\right)$$

$$\frac{V_2}{V_1} = \frac{(nRT/p_2)}{(nRT/p_1)} = \frac{p_1}{p_2}$$

$$w = nRT \ln\left(\frac{p_2}{p_1}\right)$$

☆断熱変化　$dq = 0$

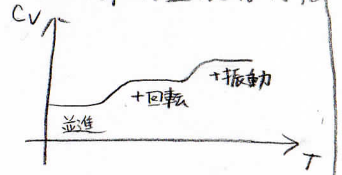
$$\begin{cases} dU = dw = -pdV \\ dU = C_v dT \end{cases} \Rightarrow -\frac{nRT}{V} dV = C_v dT$$



$$-nR\int_{V_1}^{V_2} \frac{dV}{V} = \int_{T_1}^{T_2} C_v \frac{dT}{T}$$

$$-nR \ln\left(\frac{V_2}{V_1}\right) = C_v \ln\left(\frac{T_2}{T_1}\right)$$

C_vは本来、温度依存する



ここでは簡単のため、定数とする

両辺の exp をとると、

$$\left(\frac{V_1}{V_2}\right)^{nR} = \left(\frac{T_2}{T_1}\right)^{C_v}$$

両辺に $V_2^{nR} T_1^{C_v}$ をかけると

$$T_1^{C_v} V_1^{nR} = T_2^{C_v} V_2^{nR} = (\text{一定})$$

両辺を $1/(nR)$ 乗する

$$V_1 T_1^{\frac{C_v}{nR}} = V_2 T_2^{\frac{C_v}{nR}} = (\text{一定})$$

モル熱容量

$$\begin{cases} C_{V,m} \equiv C_V / n \\ C_{p,m} \equiv C_p / n \end{cases}$$

Mayerの式 $C_{p,m} - C_{V,m} = R$ より.

$$\frac{C_V}{nR} = \frac{C_{V,m}}{C_{p,m} - C_{V,m}}$$

$\gamma \equiv \dfrac{C_{p,m}}{C_{V,m}}$ とすると.

$$\frac{C_{V,m}}{C_{p,m} - C_{V,m}} = \left(\frac{C_{p,m} - C_{V,m}}{C_{V,m}} \right)^{-1}$$

$$= \frac{1}{\gamma - 1}$$

$$V T^{\frac{1}{\gamma-1}} = (一定)$$

$$\Downarrow$$

$$T V^{\gamma-1} = (一定)$$

さらに $T = \dfrac{pV}{nR}$ より.

$$p V^{\gamma} = (一定)$$

Poisson(ポアソン)の式

単原子分子気体 $\rightarrow \gamma = 5/3 = 1.66\cdots$

二原子分子気体（並進と回転）$\rightarrow \gamma = 7/5 = 1.4$